



# LINEAR ALGEBRA

Spring Semester 2014  
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<http://shannon.cm.nctu.edu.tw/la.htm>

## Homework 12 of June 05, 2014

Deadline: June 12, 2014

### Problem 1 (30%)

### *Singular Value Decomposition*

Answer the following questions for matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ .

- (10%) Find its eigenvalues and eigenvectors.
- (10%) Find its Jordan form.
- (10%) Find its singular value decomposition (SVD).

### Problem 2 (20%)

### *Properties of Similar Matrices*

True or false? Explain if true, or find a counterexample if false:

- (5%) An  $n \times n$  matrix  $A$  is similar to  $A$  itself.
- (5%) If  $A$  is similar to  $B$  and  $B$  is similar to  $C$ , then  $A$  is similar to  $C$ .
- (5%) If  $A$  is similar to  $B$ , then they have the same eigenvalues.
- (5%) If  $A^2$  is similar to  $B^2$ , then  $A$  is similar to  $B$ .

### Problem 3 (20%)

### *Singular Values*

Suppose  $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ .

- (10%) Find all singular values of  $A$ .
- (10%) Determine  $e^{tA}$ .  
Hint: What is  $A^n$  for  $n \geq 3$ ?

### Problem 4 (30%)

### *Positive Definite*

Suppose  $A$  is a real  $m$  by  $n$  Matrix.

- (10%) Prove that the symmetric matrix  $A^T A$  satisfies that

$$\mathbf{x}^T A^T A \mathbf{x} \geq 0$$

for every non-zero vector  $\mathbf{x}$  in  $\mathbb{R}^n$ .

- (10%) According to (a), under what condition on  $A$  is  $A^T A$  positive definite?
- (10%) Prove that  $A^T A$  is not positive definite if  $m < n$ .