LINEAR ALGEBRA

Spring Semester 2014
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Midterm Exam 1 of 27 March, 2014

Problem 1 (15%) ***Cauchy–Schwarz Inequality***

For positive real numbers a, b, c with $a + b + c = 1$, use Cauchy-Schwarz Inequality to show that

$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2+\left(c+\frac{1}{c}\right)^2 \geq \frac{100}{3}.$$

Hint: Use Cauchy-Schwarz Inequality

$$\|\mathbf{x}\|^2\|\mathbf{y}\|^2 \geq |\mathbf{x}\cdot\mathbf{y}|^2$$

with $\mathbf{x} = \begin{bmatrix} a+\frac{1}{a} \\ b+\frac{1}{b} \\ c+\frac{1}{c} \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Then use Cauchy-Schwarz inequality again to find a lower bound to $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.

Problem 2 (15%) ***Right Inverse of a Non-Square Matrix***

Find the right inverse

$$\mathsf{R}_{4\times 3} = \begin{bmatrix} & \mathbf{r}_1' & \\ & \mathbf{r}_2' & \\ & \mathbf{r}_3' & \\ 1 & 1 & 1 \end{bmatrix}$$

of a non-square matrix

$$\mathsf{A}_{3\times 4} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 3 & 1 \\ 2 & 1 & 4 & 3 \end{bmatrix}$$

such that AR is equal to the 3 by 3 identity matrix $\mathsf{I}_{3\times 3}$.
Hint: Find rref(A).

Problem 3 (12%) $\mathsf{PA} = \mathsf{LU}$ ***Factorization***

Find the $\mathsf{PA} = \mathsf{LU}$ factorization for

$$\mathsf{A} = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & 9 \\ 1 & 1 & 3 \end{bmatrix}.$$

Problem 4 (16%) ***Properties of Inverses***

True or False (with a counterexample if false and a proof if true):

a) (4%) If A is an invertible tridiagonal matrix, then A^{-1} is a tridiagonal matrix.

b) (4%) If A, B and C are all n by n matrices and $\mathsf{AB}+\mathsf{AC}=\mathsf{I}$, then A is invertible.

c) (4%) If both A and B are $n \times n$ and invertible, then $\mathsf{A}+\mathsf{B}$ is invertible.

d) (4%) If A is an invertible matrix and c is a non-zero real number, then $(c\mathsf{A})^{-1} = \frac{1}{c}\mathsf{A}^{-1}$.

Problem 5 (18%) — *Inverse of Matrix*

a) (12%) Find the inverse of the matrix

$$\mathsf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}.$$

b) (6%) Solve the linear equations

$$\begin{aligned} x + 2y + 3z &= 0 \\ 3x + 2y + 2z &= 2 \\ 2x + y + z &= -1. \end{aligned}$$

Problem 6 (24%) — UL *Factorization*

Give a matrix A as

$$\mathsf{A} = \begin{bmatrix} 1 & 4 & 1 \\ 4 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

a) (12%) Solve $\mathsf{A} = \mathsf{UL}$ factorization, where U is an upper triangular matrix and L is an lower triangular matrix.

Hint: Do "backward" elimination first instead of "forward" elimination.

b) (6%) Let

$$\boldsymbol{b} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$

Solve $\mathsf{U}\boldsymbol{c} = \boldsymbol{b}$.

c) (6%) Solve $\mathsf{A}\boldsymbol{x} = \boldsymbol{b}$.