Notes for the 2nd Midterm

Please note that the 2nd midterm is an Open Book test.

Chapter 10 (except Section 10-3)

1. Whether sampling a continuous WSCS process gives a discrete WSCS process
2. Theorems 10-9 and 10-10
3. Random sampling for deterministic signal $x(t)$
4. Tapped delay line approximation of matched filter
5. Smoothing in the MS sense

   Some people may be dubious about (or have troubles to understand) how we can take partial derivative onto e with respect to $h(v)$. Here, I provide an alternative approach to determine the optimal $h_{\text{opt}}(v)$.

   Recall that we wish to minimize e subject to $\int_{-T}^{T} h(\tau)d\tau = 1$ and $\int_{-T}^{T} \tau^2 h(\tau)d\tau = c$. By using the Lagrange multipliers techniques, it becomes to minimize

   $$\begin{aligned} e &= \frac{[f''(t_0)]^2}{4}c^2 + \frac{N_0}{2}\int_{-T}^{T} h^2(\tau)d\tau - \lambda_1\left(\int_{-T}^{T} h(\tau)d\tau - 1\right) \\ &\quad -\lambda_2\left(\int_{-T}^{T} \tau^2 h(\tau)d\tau - c\right) \\ &= \int_{-T}^{T}\left(\frac{N_0}{2}h^2(\tau) - \left(\lambda_1 + \lambda_2\tau^2\right)h(\tau)\right)d\tau + \frac{[f''(t_0)]^2}{4}c^2 + \lambda_1 + \lambda_2 c \\ &= \int_{-T}^{T}\left(\frac{N_0}{2}\left[h(\tau) - \frac{(\lambda_1 + \lambda_2\tau^2)}{N_0}\right]^2 - \frac{(\lambda_1 + \lambda_2\tau)^2}{2N_0}\right)d\tau \\ &\quad + \frac{[f''(t_0)]^2}{4}c^2 + \lambda_1 + \lambda_2 c \end{aligned}$$

   Apparently, choosing $h(\tau)$ other than $\frac{(\lambda_1+\lambda_2\tau^2)}{N_0}$ can only increase e. Thus, $h_{\text{opt}}(\tau) = \frac{(\lambda_1+\lambda_2\tau^2)}{N_0}$ minimizes e.

6. Applications of Poisson sum formula

Chapter 11

1. Determination of innovation filters for both analog and discrete regular processes

2. Yule-Walker Equations, in particular for Example 11-7, ask yourself

   - what will happen when $b_0 \neq 0$ and $|a_1| > 1$?
   - what will happen when $b_0 \neq 0$ and $|a_1| = 1$?
   - what will happen when $b_0 \neq 0$ and $|a_1| < 1$?
   - what will happen when $b_0 = 0$ and $|a_1| > 1$?
   - what will happen when $b_0 = 0$ and $|a_1| = 1$?
   - what will happen when $b_0 = 0$ and $|a_1| < 1$?

3. The theorem in Slide 11-56 and also Theorem 11-1

4. Fourier Series and Karhunen-Loève Expansions

   - Eigenvalues and eigenfunctions of a non-stationary white process with autocorrelation function $R(t, s) = q(t)\delta(t - s)$ over $[0, T)$.

5. Wold's decomposition

   - Since $E[|\boldsymbol{y}[t]|^2] = 0$, can we say $1 - A[z]W[z] = 0$?

6. The subjects below will be excluded from this example: (i) windowing filters, (ii) Fourier-Stieltjes representation of WSS processes and (iii) Bispectra and Third Order Moments.