Sample problems for the second quiz

1. (a) Find the "extended inverse Fourier transform" of $F(\omega) = 1$ through

$$\lim_{n\to\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{-|\omega|/n} e^{j\omega t} d\omega.$$

(b) Find the "extended inverse Fourier transform" of $F(\omega) = j\omega$ through

$$\lim_{n\to\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{-|\omega|/n} e^{j\omega t} d\omega.$$

(c) Find the Fourier transforms of the extended inverse Fourier transforms in (a) and (b), respectively. Are they equal to 1 and $j\omega$ in (a) and (b), respectively? Please comment on your answers.

2. (a) Show that $\boldsymbol{x}(t) = \sum_{n=-\infty}^{\infty} \boldsymbol{c}_n g(t - nT)$ is wide-sense cyclostationary if $\{\boldsymbol{c}_n\}_{n=-\infty}^{\infty}$ is zero-mean wide-sense stationary (i.e., $R_{cc}[n, m] = R_{cc}[n - m]$).

Hint: A random process $\boldsymbol{x}(t)$ *is* wide-sense cyclostationary stationary *(WSCS) with period* T *if* $\eta_{xx}(t+mT) = \eta_{xx}(t)$ *and* $R_{xx}(t_1+mT, t_2+mT) = R_{xx}(t_1, t_2)$ *for every integer* m.

(b) Is $\boldsymbol{x}(t)$ in (a) MS periodic? Justify your answer.

Hint: A process $\boldsymbol{x}(t)$ *is* MS periodic *if, and only if, its autocorrelation function is* doubly periodic, *namely,*

$$R_{xx}(t_1 + mT, t_2 + nT) = R_{xx}(t_1, t_2) \textit{ for every integer } m \textit{ and } n.$$

(c) Show that $\boldsymbol{x}(t) = \sum_{n=-\infty}^{\infty} \boldsymbol{c}_n g(t - nT)$ is wide-sense cyclostationary if $\{\boldsymbol{c}_n\}_{n=-\infty}^{\infty}$ is zero-mean wide-sense cyclostationary with period M (i.e., $R_{cc}[n, m] = R_{cc}[n + kM, m + kM]$ for every k). What is the period of the wide-sense cyclostationarity of $\boldsymbol{x}(t)$?

3. (a) If $\boldsymbol{x}(t)$ is BL, then

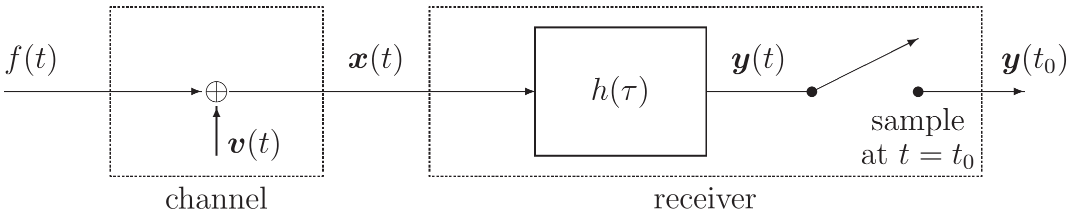
$$\lim_{w\to\infty} \frac{1}{2w} \int_{-w}^{w} E\left[|\boldsymbol{x}(t + \tau) - \boldsymbol{x}(t)|^2\right] dt \le \sigma^2\tau^2 \bar{R}_{xx}(0),$$

provided the limit exists.

(b) Show that we can improve the upper bound to:

$$\lim_{w\to\infty} \frac{1}{2w} \int_{-w}^{w} E\left[|\boldsymbol{x}(t + \tau) - \boldsymbol{x}(t)|^2\right] dt \le \min\{\sigma^2\tau^2, 4\} \cdot \bar{R}_{xx}(0).$$

4.



Suppose $h(\tau)$ satisfies that $h(\tau) = 0$ for $|\tau| > T$, $h(-\tau) = h(\tau)$, and $\int_{-T}^{T} h(\tau)d\tau = 1$. Assume that $\boldsymbol{v}(t)$ is zero-mean white with PSD $S_{vv}(\omega) = \frac{N_0}{2}$.

(a) Subject to $f(t_0 - \tau) = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3$, where $a_0 = f(t_0)$, find the filter $h(\tau)$ that minimizes $E\{[\boldsymbol{y}(t_0) - f(t_0)]^2\}$.

(b) Subject to $f(t_0 - \tau) = a_0 + a_4\tau^4$, where $a_0 = f(t_0)$, find the filter $h(\tau)$ that minimizes $E\{[\boldsymbol{y}(t_0) - f(t_0)]^2\}$.