2006 First Quiz for Random Processes

Name:____________________ **Student ID:**____________________

1. (50%) Define a random process $\{\boldsymbol{x}(t), t \in \Re\}$ that is defined over the probability space $(S, \mathcal{F}, P)$ as:

$$\begin{aligned} \boldsymbol{x}(t, \blacktriangle) &= t \bmod 2 \\ \boldsymbol{x}(t, \blacktriangledown) &= (t+1) \bmod 2 \end{aligned}$$

where $S = \{\blacktriangle, \blacktriangledown\}$, $\mathcal{F}$ is the set of all subsets of S, and P satisfying $P(\{\blacktriangle\}) = P(\{\blacktriangledown\}) = 0.5$.

   (a) Determine $\Pr\{\boldsymbol{x}(0.5) \le 1 \text{ and } \boldsymbol{x}(1.5) \le 1\}$.

   (b) Determine $\Pr\{\min_{0.5 \le t \le 1.5} \boldsymbol{x}(t) \le 1\}$.

2. (50%) For a given probability space $(S, \mathcal{F}, P)$, where

$$\begin{aligned} S &= \{\blacktriangle, \blacktriangledown, \square, \blacksquare, \Diamond, \blacklozenge\} \\ \mathcal{F} &= \{\emptyset, \{\blacktriangle, \blacktriangledown, \square\}, \{\blacksquare, \Diamond, \blacklozenge\}, S\} \\ P &= \{0, 1/2, 1/2, 1\} \end{aligned}$$

   (a) define a legitimate non-deterministic random variable $\boldsymbol{y}$ that is well-defined over the given probability space,

   (b) and also, calculate $E[\boldsymbol{y}]$.

Solutions

1.

   (a) Since

$$\boldsymbol{x}(0.5, \blacktriangle) = 0.5 \bmod 2 = 0.5 \le 1$$

$$\textbf{but } \boldsymbol{x}(1.5, \blacktriangle) = 1.5 \bmod 2 = 1.5 > 1,$$

   and

$$\boldsymbol{x}(0.5, \blacktriangledown) = (0.5+1) \bmod 2 = 1.5 > 1$$

$$\textbf{although } \boldsymbol{x}(1.5, \blacktriangledown) = (1.5+1) \bmod 2 = 0.5 \le 1,$$

none of the elements in S is contained in $\{\zeta \in S : \boldsymbol{x}(0.5, \zeta) \leq 1 \text{ and } \boldsymbol{x}(1.5, \zeta) \leq 1\}$.
Hence,

$$\begin{aligned} &\Pr\{\boldsymbol{x}(0.5) \leq 1 \text{ and } \boldsymbol{x}(1.5) \leq 1\} \\ &= P\left(\{\zeta \in S : \boldsymbol{x}(0.5, \zeta) \leq 1 \text{ and } \boldsymbol{x}(1.5, \zeta) \leq 1\}\right) \\ &= P(\emptyset) = 0. \end{aligned}$$

(b) Since

$$\min_{0.5 \leq t \leq 1.5} \boldsymbol{x}(t, \blacktriangle) = \min_{0.5 \leq t \leq 1.5} (t \bmod 2) = 0.5 \leq 1$$

and

$$\min_{0.5 \leq t \leq 1.5} \boldsymbol{x}(t, \blacktriangledown) = \min_{0.5 \leq t \leq 1.5} ((t+1) \bmod 2) = 0.5 \leq 1$$

both elements in S is contained in $\{\zeta \in S : \min_{0.5 \leq t \leq 1.5} \boldsymbol{x}(t, \zeta) \leq 1\}$.
Hence,

$$\begin{aligned} &\Pr\left\{\min_{0.5 \leq t \leq 1.5} \boldsymbol{x}(t) \leq 1\right\} \\ &= P\left(\left\{\zeta \in S : \min_{0.5 \leq t \leq 1.5} \boldsymbol{x}(t, \zeta) \leq 1\right\}\right) \\ &= P(S) = 1. \end{aligned}$$

2. An exemplified answer can be found in slide 9-20.