2012 First Quiz for Random Processes

Name ______________ **Student ID** ______________

1. Suppose the probability space considered is

$$(S = \{\oplus, \ominus, \otimes, \oslash\}, \mathcal{F} = 2^S, P = \{p_1, p_2, p_3, p_4\} \text{ resp. for } S).$$

At some specific time t, we have

$$\boldsymbol{x}(t,\oplus) = \boldsymbol{x}(t,\ominus) = 1 \quad \text{and} \quad \boldsymbol{x}(t,\otimes) = \boldsymbol{x}(t,\oslash) = -1,$$

and the memoryless $\boldsymbol{T}$ satisfies

$$\boldsymbol{T}(1,\oplus) = 1, \quad \boldsymbol{T}(1,\ominus) = -1, \quad \boldsymbol{T}(-1,\otimes) = 1 \quad \text{and} \quad \boldsymbol{T}(-1,\oslash) = -1.$$

(a) (50%) Find $\boldsymbol{y}(t,\oplus)$, $\boldsymbol{y}(t,\ominus)$, $\boldsymbol{y}(t,\otimes)$, $\boldsymbol{y}(t,\oslash)$.

(b) (50%) Prove for $a, b \in \{-1, 1\}$, it is always true that

$$\Pr[\boldsymbol{x}(t) = a \wedge \boldsymbol{T}(a) = b] = \Pr[\boldsymbol{x}(t) = a] \times \Pr[\boldsymbol{T}(a) = b]$$

Hint:

$$\Pr[\boldsymbol{T}(a) = b] = \Pr[\boldsymbol{y}(t) = b | \boldsymbol{x}(t) = a]$$

and

$$\Pr[\boldsymbol{x}(t) = a \wedge \boldsymbol{T}(a) = b] = \Pr[\boldsymbol{x}(t) = a \wedge \boldsymbol{y}(t) = b].$$

Solution.

1. (a)

$$\begin{aligned}
y(t,\oplus) &= \boldsymbol{T}(x(t,\oplus),\oplus) = \boldsymbol{T}(1,\oplus) = 1\\
y(t,\ominus) &= \boldsymbol{T}(x(t,\ominus),\ominus) = \boldsymbol{T}(1,\ominus) = -1\\
y(t,\otimes) &= \boldsymbol{T}(x(t,\otimes),\otimes) = \boldsymbol{T}(-1,\otimes) = 1\\
y(t,\oslash) &= \boldsymbol{T}(x(t,\oslash),\oslash) = \boldsymbol{T}(-1,\oslash) = -1
\end{aligned}$$

(b)

$$\begin{aligned}
\Pr[\boldsymbol{x}(t) = a \wedge \boldsymbol{T}(a) = b] &= \Pr[\boldsymbol{x}(t) = a \wedge \boldsymbol{y}(t) = b]\\
&= \Pr[\boldsymbol{y}(t) = b | \boldsymbol{x}(t) = a] \times \Pr[\boldsymbol{x}(t) = a]\\
&= \Pr[\boldsymbol{T}(a) = b] \times \Pr[\boldsymbol{x}(t) = a]
\end{aligned}$$