2007 Second Quiz for Random Processes

Name:__________________ **Student ID:**__________________

1. Let $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ be the input and output of a system, where $\boldsymbol{y}(t) = \boldsymbol{x}(t - \boldsymbol{\theta})$ for some random variable $\boldsymbol{\theta}$ independent of $\boldsymbol{x}(t)$.
   (a) (25%) What is the transfer function of the system?
   (b) (25%) Is it a linear system? Justify your answer.
   (c) (25%) Is it a time-invariant system? Justify your answer.
   (d) (25%) Prove that "unsynchronization" defined above does not change the time-average power spectrum density. (Note: You may directly use the Fundamental Theorems and Theorems 9-2 and 9-4.)

Solutions

1. $\boldsymbol{H}(\omega) = e^{-j\omega\boldsymbol{\theta}}$.
2. Yes, since $\boldsymbol{Y}_j(\omega) = \boldsymbol{X}_j(\omega)\boldsymbol{H}(\omega)$ implies $\sum_j \boldsymbol{Y}_j(\omega) = \sum_j [\boldsymbol{X}_j(\omega)\boldsymbol{H}(\omega)] = \left[\sum_j \boldsymbol{X}_j(\omega)\right] \boldsymbol{H}(\omega)$.
3. Yes, since the statistics of $\boldsymbol{H}(\omega)$ does not change with time.
4. $\bar{S}_{yy}(\omega) = E\left[|\boldsymbol{H}(\omega)|^2 \bar{S}_{xx}(\omega)\right] = E\left[|\boldsymbol{H}(\omega)|^2\right] \bar{S}_{xx}(\omega) = \bar{S}_{xx}(\omega)$.