Sample Problems for the Quiz on 22\textsuperscript{th} Dec.

13.1 The scattering function $S(\tau; \lambda)$ for a fading multipath channel is nonzero for the range of values $0 \leq \tau \leq 1$ ms and $-0.1$ Hz $\leq \lambda \leq 0.1$ Hz. Assume that the scattering function is approximately uniform in the two variables.

(a) Give numerical values for the following parameters:

(i) The multipath spread of the channel.

(ii) The Doppler spread of the channel.

(iii) The coherence time of the channel.

(iv) The coherence bandwidth of the channel.

(v) The spread factor of the channel.

(b) Explain the meaning of the following, taking into consideration the answers given in (a):

(i) The channel is frequency-nonselective.

(ii) The channel is slowly fading.

(iii) The channel is frequency-selective.

13.3 Suppose that the binary signal $\pm s_1(t)$ is transmitted over a fading channel and the received signal is

$$r_1(t) = \pm a s_1(t) + z(t), \quad 0 \leq t \leq T$$

where $z(t)$ is zero-mean white Gaussian noise with autocorrelation function

$$R_{zz}(\tau) = 2N_0 \delta(\tau)$$

The energy in the transmitted signal is $E = \frac{1}{2} \int_0^T |s_1(t)|^2 \, dt$. The channel gain $a$ is specified by the probability density function

$$p(a) = 0.18(a) + 0.98(a - 2)$$

(a) Determine the average probability of error $P_b$ for the demodulator that employs a filter matched to $s_1(t)$.

(b) What value does $P_b$ approach as $E/N_0$ approaches infinity?

(c) Suppose that the same signal is transmitted on two statistically independently fading channels with gains $a_1$ and $a_2$, where

$$p(a_k) = 0.18(a_k) + 0.98(a_k - 2), \quad k = 1, 2$$

The noises on the two channels are statistically independent and identically distributed. The demodulator employs a matched filter for each channel and simply adds the two filter outputs to form the decision variable. Determine the average $P_b$.

(d) For the case in (c) what value does $P_b$ approach as $E/N_0$ approaches infinity?

13.4 A multipath fading channel has a multipath spread of $T_m = 1$ s and a Doppler spread $B_d = 0.01$ Hz. The total channel bandwidth at bandpass available for signal transmission is $W = 5$ Hz. To reduce the effects of intersymbol interference, the signal designer selects a pulse duration $T = 10$ s.

(a) Determine the coherence bandwidth and the coherence time.

(b) Is the channel frequency selective? Explain.

(c) Is the channel fading slowly or rapidly? Explain.