13.2 Consider a binary communication system for transmitting a binary sequence over a fading channel. The modulation is orthogonal FSK with third-order frequency diversity (\( L = 3 \)). The demodulator consists of matched filters followed by square-law detectors. Assume that the FSK carriers fade independently and identically according to a Rayleigh envelope distribution. The additive noises on the diversity signals are zero-mean Gaussian with autocorrelation functions \( E[z_k(t)z_k(t+r)] = 2N_0\delta(r) \). The noise processes are mutually statistically independent.

- The transmitted signal may be viewed as binary FSK with square-law detection, generated by a repetition code of the form

\[
1 \rightarrow c_1 = [1 \ 1 \ 1], \quad 0 \rightarrow c_0 = [0 \ 0 \ 0]
\]

Determine the error rate performance \( P_{bh} \) for a hard-decision decoder following the square-law-detected signals.

- Evaluate \( P_{bh} \) for \( \gamma_c = 100 \) and 1000.

- Evaluate the error rate \( P_{bh} \) for \( \gamma_c = 100 \) and 1000 if the decoder employs soft-decision decoding.

- Consider the generalization of the result in (a). If a repetition code of block length \( L \) (\( L \) odd) is used, determine the error probability \( P_{bh} \) of the hard-decision decoder and compare that with \( P_{bh} \), the error rate of the soft-decision decoder. Assume \( \gamma_c \gg 1 \).

13.6 Consider the model for a binary communication system with diversity as shown in Figure P13.6. The channels have fixed attenuations and phase shifts. The \( \{z_k(t)\} \) are complex-valued white Gaussian noise processes with zero-mean and autocorrelation functions

\[
R_z(t) = E\left[z_k(t)z_k(t+r)\right] = 2N_0\delta(r)
\]

(Note that the spectral densities \( \{N_0\} \) are all different.) Also, the noise processes \( \{z_k(t)\} \) are mutually statistically independent. The \( \{\beta_k\} \) are complex-valued weighting factors to be determined. The decision variable from the combiner is

\[
U = \text{Re}\left(\sum_{k=1}^{L} \beta_k U_k\right)
\]

- Determine the PDF \( p(u) \) when +1 is transmitted.

- Determine the probability of error \( P_e \) as a function of the weights \( \{\beta_k\} \).

- Determine the values of \( \{\beta_k\} \) that minimize \( P_e \).

13.10 A DS spread spectrum system is used to resolve the multipath signal components in a two-path radio signal propagation scenario. If the path length of the secondary path is 300 m longer than that of the direct path, determine the minimum chip rate necessary to resolve the multipath components.