Sample Problems for the Quiz on 24th Nov

1.) (problem 4.36)

In on-off keying of a carrier modulated signal, the two possible signals are

\[ s_0(t) = 0, \quad s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t, \quad 0 \leq t \leq T_b \]

The corresponding received signals are

\[ r(t) = n(t), \quad 0 \leq t \leq T_b \]
\[ r(t) = \sqrt{\frac{2E_b}{T_b}} \cos (2\pi f_c t + \phi) + n(t), \quad 0 \leq t \leq T_b \]

where \( \phi \) is the carrier phase and \( n(t) \) is AWGN.

1. Sketch a block diagram of the receiver (demodulator and detector) that employs non-coherent (envelope) detection.
2. Determine the PDFs for the two possible decision variables at the detector corresponding to the two possible received signals.
3. Derive the probability of error for the detector.

2.) (problem 4.50)

A fading channel can be represented by the vector channel model \( \mathbf{r} = \mathbf{a}s_m + \mathbf{n} \), where \( \mathbf{a} \) is a random variable denoting the fading, whose density function is given by the Rayleigh distribution

\[ p(a) = \begin{cases} 2ae^{-a^2} & a > 0 \\ 0 & a < 0 \end{cases} \]

1. Assuming that equiprobable signals, binary antipodal signaling, and coherent detection are employed, what is the structure of the optimal receiver?
2. Show that the bit error probability in this case can be written as

\[ P_b = \frac{1}{2} \left( 1 - \sqrt{\frac{E_b/N_0}{1 + E_b/N_0}} \right) \]

and for large SNR values we have

\[ P_b \approx \frac{1}{4E_b/N_0} \]

3. Assuming an error probability of \( 10^{-3} \) is desirable, determine the required SNR per bit (in dB) if (i) the channel is nonfading and (ii) the channel is a fading channel. How much more power is required by the fading channel to achieve the same bit error probability?

4. Show that if binary orthogonal signaling and noncoherent detection are employed, we have

\[ P_b = \frac{1}{2 + E_b/N_0} \]
3.)

The steady-state trellis of a code is shown as below, when the received bits are (10, 10, 01, 11, ...), please plot the four survival paths after receiving the first eight bits (assume initial state is “00”).

4.) (problem 5.8)

Based on an ML criterion, determine a carrier phase estimation method for binary on-off keying modulation.