Problem 13.1

Based on the info about the scattering function we know that the multipath spread is \( T_m = 1 \text{ ms} \), and the Doppler spread is \( B_d = 0.2 \text{ Hz} \).

(a) (i) \( T_m = 10^{-3} \text{ sec} \)
(ii) \( B_d = 0.2 \text{ Hz} \)
(iii) \( (\Delta t)_c \approx \frac{1}{f_c} = 5 \text{ sec} \)
(iv) \( (\Delta f)_c \approx \frac{1}{f_m} = 1000 \text{ Hz} \)
(v) \( T_m B_d = 2 \cdot 10^{-4} \)

(b) (i) Frequency non-selective channel: This means that the signal transmitted over the channel has a bandwidth less than 1000 Hz.
(ii) Slowly fading channel: The signaling interval \( T \) is \( T \ll (\Delta t)_c \).
(iii) The channel is frequency selective: the signal transmitted over the channel has a bandwidth greater than 1000 Hz.

Problem 13.3

(a) For a fixed channel, the probability of error is: \( P_e(a) = Q\left(\sqrt{\frac{2a^2}{N_0}}\right) \). We now average this conditional error probability over the possible values of \( a \), which are \( a = 0 \), with probability 0.1, and \( a = 2 \) with probability 0.9. Thus:

\[
P_e = 0.1Q(0) + 0.9Q\left(\sqrt{\frac{4}{N_0}}\right) = 0.05 + 0.9Q\left(\sqrt{\frac{8}{N_0}}\right)
\]

(b) As \( \frac{\xi}{N_0} \to \infty \), \( P_e \to 0.05 \)

(c) When the channel gains \( a_1, a_2 \) are fixed, the probability of error is:

\[
P_e(a_1, a_2) = Q\left(\sqrt{\frac{2(a_1^2 + a_2^2)}{N_0}}\right)
\]

Averaging over the probability density function \( p(a_1, a_2) = p(a_1) \cdot p(a_2) \), we obtain the average probability of error:

\[
P_e = (0.1)^2Q(0) + 2 \cdot 0.9 \cdot 0.1 \cdot Q\left(\sqrt{\frac{8}{N_0}}\right) + (0.9)^2Q\left(\sqrt{4\frac{\xi}{N_0}}\right)
\]

\[
= 0.005 + 0.18Q\left(\sqrt{\frac{8}{N_0}}\right) + 0.81Q\left(\sqrt{\frac{4\xi}{N_0}}\right)
\]

(d) As \( \frac{\xi}{N_0} \to \infty \), \( P_e \to 0.005 \)
Problem 13.4

(a)

\[ T_m = 1 \text{ sec} \Rightarrow (\Delta f)_c \approx \frac{1}{T_m} = 1 \text{ Hz} \]
\[ B_d = 0.01 \text{ Hz} \Rightarrow (\Delta t)_c \approx \frac{1}{B_d} = 100 \text{ sec} \]

(b) Since \( W = 5 \text{ Hz} \) and \( (\Delta f)_c \approx 1 \text{ Hz} \), the channel is frequency selective.

(c) Since \( T=10 \text{ sec} < (\Delta t)_c \), the channel is slowly fading.