Lecture Notes in Information Theory
Volume II: Problems

Po-Ning Chen† and Fady Alajaji‡

† Department of Communication Engineering
National Chao-Tung University
1001, Ta-Hsueh Road
Hsin Chu, Taiwan 30050
Republic of China
Email: poning@cc.nctu.edu.tw

‡ Department of Mathematics & Statistics,
Queen’s University, Kingston, ON K7L 3N6, Canada
Email: fady@polya.mast.queensu.ca

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Chapter 3

Problems

3-1 Let $U_1, U_2, \ldots$ be an i.i.d. binary sequence with marginal distribution

$$P_U(0) = P_U(1) = \frac{1}{2}.$$ 

Define the source $X_1, X_2, \ldots$ by

$$X_n = U_1 \times U_2 \times \cdots \times U_n.$$ 

Find the sup-entropy rate and inf-entropy rate of the source $X = \{X^n = (X_1, \ldots, X_n)\}$. 

3-2 Show that when $t \to 0$, the exponential cost function

$$L(t) \triangleq \frac{1}{t} \log_2 \left( \sum_{x \in \mathcal{X}} P_X(x) 2^{t \ell(x)} \right)$$

reduce to

$$L(0) = \log_2(e) \cdot \sum_{x \in \mathcal{X}} P_X(x) \ell(x),$$

which is the average codeword length. Also show that in the case of $t \to \infty$,

$$L(\infty) = \max_{x \in \mathcal{X}} \ell(x),$$

which is the maximum codeword length for all binary codewords.
Chapter 4

Problems

4-1 Prove that the worst-case complexity of a random variable $X$ is no less than $\lceil R(X) \rceil$. Also prove that the average-case complexity of a random variable $X$ lies between $H(X)$ and $H(X) + 1$.

(Hint: To write an algorithm with desired worst-case complexity for source $X$ is similar to establish a binary data compaction block code with desired codeword length for source $X$. Likewise, to write an algorithm with desired average-case complexity for random variable $X$ is similar to design a binary variable-length data compaction code with desired average codeword length for source $X$.)