Give a random variable $Z$ with zero-mean and finite variance. It can be shown similarly as slide 28-20 that

$$
\varphi_Z(t) - 1 = \int_{-\infty}^{\infty} \left(e^{itx} - 1 - itx\right) \frac{1}{x^2} \mu(dx),
$$

where

$$
\mu(-\infty, x] = \int_{-\infty}^{x} z^2 dF_Z(z),
$$

and

$$
\varphi_Z(t) = \int_{-\infty}^{\infty} e^{itz} dF_Z(z).
$$

Now how to find the finite-measure $\mu$ corresponding to random $X$ of zero-mean and finite variance?

1. Firstly, derive

$$
\varphi_X(t) = \int_{-\infty}^{\infty} e^{itz} dF_X(x).
$$

2. Secondly, derive

$$
\varphi_Z(t) = \log \varphi_X(t) + 1.
$$

3. Thirdly, note that $\varphi_Z(0) = 1$ and $\varphi_Z(t)$ is (uniformly) continuous if $\varphi_X(t)$ is (uniformly) continuous.

4. Fourthly, by

$$
\varphi'_Z(t) = \frac{\varphi'_X(t)}{\varphi_X(t)}
$$

and

$$
\varphi''_Z(t) = \frac{\varphi''_X(t) \varphi_X(t) - [\varphi'_X(t)]^2}{\varphi_X^2(t)},
$$

we obtain $Z$ has zero-mean and finite variance.

5. Fifthly, find $dF_Z(z)$ according to $\varphi_Z(t)$.

6. Lastly,

$$
\mu(-\infty, x] = \int_{-\infty}^{x} z^2 dF_Z(z).
$$